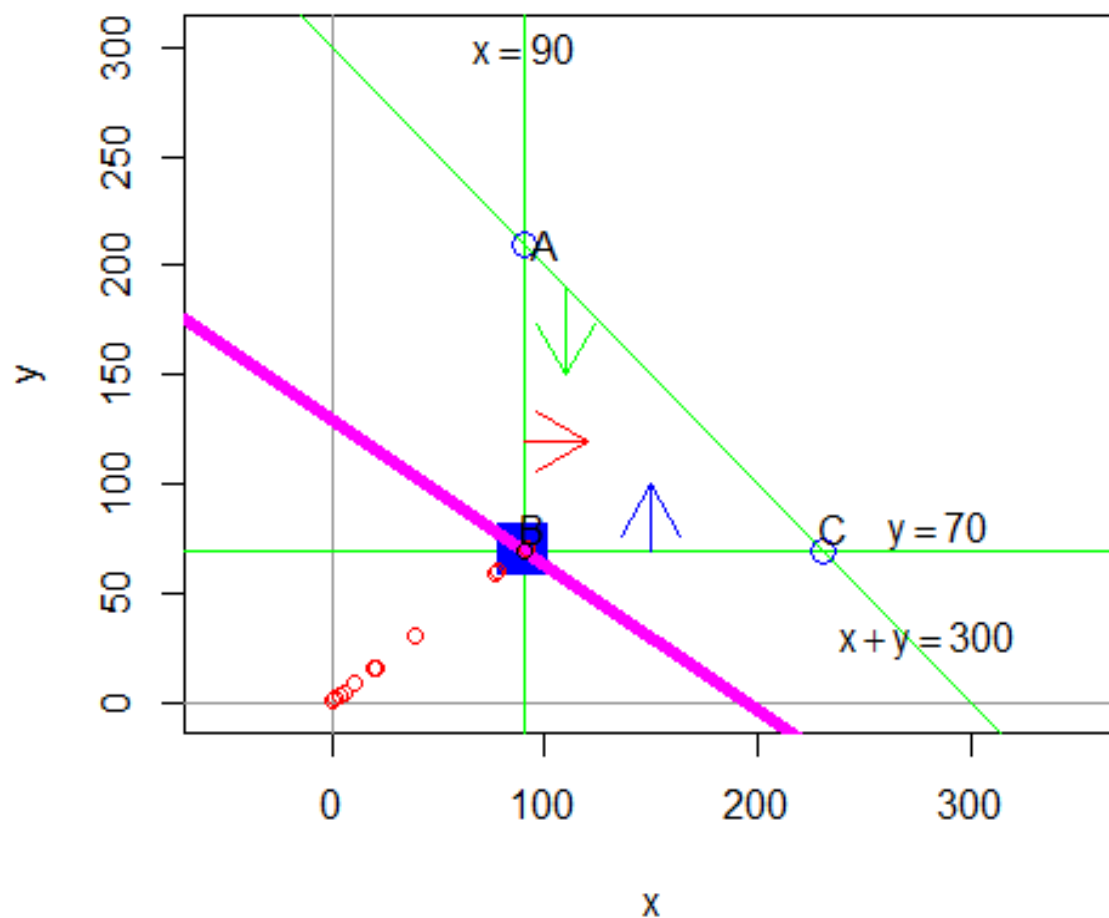


Henry De-Graft Acquah

# Introduction to Linear Programming for Economic Analysis



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Berichte aus der Mathematik

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**Introduction to Linear Programming for Economic Analysis**

Shaker Verlag  
Aachen 2018

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

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Printed in Germany.

ISBN 978-3-8440-5967-0

ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

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## **Dedication**

**To Nanny, Jeremy and Joyce**



# Preface

On assumption of duty at the University of Cape Coast in July 2008, I had the opportunity to teach Quantitative and Research methods. In the ensuing years, I also taught Computer applications in Agricultural Economics, Farm Management and Production Economics, and Applied Statistics and Econometrics. While teaching these quantitative courses to graduate students, the only thing I found lacking was an introductory textbook, one that provides students and professionals in the social sciences and related fields with a fundamental understanding of how to use linear programming techniques in empirical research. Thus, there is a need for a textbook, which will explain linear programming techniques and implement them with a free and readily available statistical software. This book fills this gap in the literature.

It presents a clear and concise introduction to linear programming methods and implements application examples of various techniques in economics using the R programming language. Fundamentally, each chapter presents theoretical concepts with application examples. One major problem of students is applying linear programming techniques to empirical data. The purpose of this book is to help overcome this problem by using the free and readily available R programming language to implement application examples for a wide range of linear programming techniques. The methodology of this book and much of its content has been tested in graduate quantitative courses in the University of Cape Coast. Students have found the approaches and the content of this book extremely useful and have made valuable suggestions for improvement.

Finally, I have inevitably accumulated debts of gratitude to colleagues, students, friends and acquaintances that I am happy to acknowledge. These include the contributions of the following persons who reviewed various draft chapters of the book.



Bernard Baah-Kumi	New Mexico State University
Owusu Kwabena Dankwa	New Mexico State University
Patrick Ofosuhenne	Bowling Green State University
Abigail Lois Quansah	Ohio University
Kwabena Krah	University of Illinois, Urbana-Champaign
Francis Annan	University of Colombia in the City of New York
Thomas Torku	Middle Tennessee State University
Clement Kyei	University of Pretoria
Frederick Nyanzu	Mississippi State University

I am also grateful to 2015/2016 M. Phil Agricultural Economics graduate students at the University of Cape Coast for their helpful suggestions and comments about this material. In particular, I would like to thank Bernard Baah-Kumi, Owusu Kwabena Dankwa, Patrick Ofosuhenne and Thomas Torku for their unflinching support throughout the preparation of this book. Special thanks go to the entire Acquah family for the encouragement and prayer support that has seen this book to a successful completion. I wish to thank my dear wife, Joyce, for her enormous support and understanding during the preparation of this book. I wish to thank the staff of Shaker Verlag for their cooperation, assistance and excellent production work.

Finally, to the Most High God, be all glory, for granting exceedingly and abundantly above what I thought or imagined.

March 2018

Prof. Dr. Henry de-Graft Acquah, Cape Coast

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