

Quadrilateral Surface Mesh Generation for Animation and Simulation

Von der Fakultät für Mathematik, Informatik und Naturwissenschaften der
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Contents

| | |
|---|-----------|
| 1. Introduction | 1 |
| 2. Quadrilateral Surface Meshes | 7 |
| 2.1. Foundations | 7 |
| 2.2. Applications | 15 |
| 2.2.1. Animation | 17 |
| 2.2.2. Simulation | 19 |
| 2.3. Quality Criteria | 20 |
| 2.4. Related Work | 22 |
| | |
| I. Mixed-Integer Optimization in Geometry Processing | 29 |
| 3. Mixed-Integer Nonlinear Programming | 33 |
| 4. General Optimization Approaches | 39 |
| 4.1. Branch-and-Bound | 41 |
| 4.2. Cutting-Plane method | 46 |
| 4.3. Branch-and-Cut | 49 |
| | |
| 5. Efficient Approximation of Quadratic MI-Problems | 53 |
| 5.1. Linear Constraints | 56 |
| 5.1.1. Lagrangian Multipliers | 56 |
| 5.1.2. Elimination Approach | 57 |
| 5.2. Integer Constraints | 60 |
| 5.2.1. Direct Rounding | 60 |
| 5.2.2. Iterative Greedy Rounding | 61 |
| 5.3. Evaluation | 64 |

| | |
|--|------------|
| II. Parametrization based Quadrilateral Mesh Generation | 71 |
| 6. Integer-Grid Mappings | 77 |
| 6.1. MINLP Formulation | 79 |
| 7. Layout guided Approach | 81 |
| 7.1. Layout Parametrization | 85 |
| 7.2. Domain Optimization | 88 |
| 7.3. Evaluation | 94 |
| 8. Orientation-field guided Approach | 99 |
| 8.1. Filtering of Salient Orientations | 102 |
| 8.2. Orientation-field Generation | 103 |
| 8.3. Sizing field computation | 108 |
| 8.4. Orientation-field Parametrization | 108 |
| 8.5. Evaluation | 115 |
| 8.6. Flexibility | 124 |
| 9. Geodesic Distance Fields | 127 |
| III. Quadmesh Optimization | 145 |
| 10. Structure Optimization | 149 |
| 10.1. Grid-Preserving Operators | 151 |
| 10.2. Helices | 158 |
| 10.3. Greedy Algorithm | 162 |
| 10.4. Evaluation | 164 |
| 11. Conclusion | 167 |
| Bibliography | 171 |