

# **GPS based velocity estimation and its application to an odometer**

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# PREFACE

In the early days of satellite navigation, the Doppler shift of the received electromagnetic waves was used to determine the position of a receiver. The most widely used Doppler navigation system was the US TRANSIT system, which was operational for 30 years starting in the early 1960ies. When TRANSIT was replaced by GPS, range rate measurements were replaced by range measurements as the primary type of information used for positioning. However, the Doppler shift observables have still been there. Every GPS receiver needs to track also the frequency of the received signals in order to obtain the pseudorange observable and to decode the navigation message broadcast by the satellites.

Typically, the GPS Doppler shift observables are used to estimate the velocity of the receiver. Since May 2000, when the intentional degradation of the ephemeris data and the satellite clock dither (selective availability) had been turned off permanently, an accuracy of better than 0.4 m/s has been attainable for standalone receivers. While this may be sufficient for navigation purposes, technical applications might require higher accuracies, and indeed the feasibility of obtaining velocity estimates with mm/s accuracies has been shown using differential carrier-phase based techniques, and often involving costly dual-frequency receivers.

During the last few years, the demand for accurate, cost-effective, and nearly ubiquitous positioning capabilities has increased enormously, driven by the cell-phone and location-based-services markets, and by the automobile market. Low-cost single-frequency consumer-grade GPS receivers and high-sensitivity GPS receivers which can track even significantly attenuated signals have been developed and are now widely used. Currently, applications are emerging which require accurate information on the distance traveled by a vehicle (or other object), possibly combined with information on time and location. Fiercely discussed examples are road pricing and time/distance/location based vehicle insurance systems.

In 2004, Drs. Hartinger and Schild of (then) HSNavi approached our Department at the Graz University of Technology with a number of questions

regarding the use of low-cost, high-sensitivity GPS receivers for such applications. These questions were mainly related to developing suitable quality control procedures, weight models, and processing strategies—topics which the Department of Engineering Geodesy and Measurement Systems, and myself, had been working on before. Subsequently, HSNav financed a related research project. As a project leader I had the opportunity to come across several challenging problems which then kept me busy—and fascinated—beyond the end of the project. This monograph is a result of this fascination.

While other authors have shown highly accurate velocity estimation experimentally before, a thorough analysis which would allow predicting the attainable accuracy for any chosen circumstances, including the use of low-cost receivers with poor oscillators or intentional frequency offsets was missing. The main part of this monograph presents such a thorough analysis, and shows that indeed mm/s level velocity accuracy may be routinely attainable if the Doppler measurements are derived from the receiver's phase-lock-loop. The analysis also shows that an accuracy better than about 1 mm/s may not be attainable even with better receivers, because many external effects which cannot be modeled with sufficient accuracy act as limiting factors.

Velocity estimates with such an accuracy may have many useful applications. One of them, namely the determination of the distance traveled by a vehicle, is investigated here as an example. The theoretical investigation shows that such a GPS odometer may have errors of 0.1% or less, and easily exceeds the typically required accuracy of tachographs (4%) if most of the time a sufficient number of satellites can be tracked by the receiver.

This monograph would not have been written without the continued support, motivation, and at times a little pressure by the Department Head, Fritz Brunner. I am deeply grateful for his guidance during my time as a young University assistant and PhD student, for the freedom to find and pursue my own research interests afterwards, and for the mentoring and friendship all along. I also thank the staff of the Institute for their help, in particular Rudolf Lummerstorfer and Sandra Schmuck who have contributed to this work by helping with hardware (and with printer problems), and organizing (looked-for and sometimes obscure) literature.

The Austrian Science Fund (FWF) has granted an Erwin Schrödinger Fellowship for a one-year post-doc period with Gérard Lachapelle at the University of Calgary. I am deeply grateful for this unique opportunity; the MATLAB software used herein has its roots in that period, and my understanding of GPS has been greatly enhanced through the discussions with students, faculty, and associates—above all with Mark Petovello.

HNav is acknowledged for granting use of data and hardware, and for encouraging my interest in the present topic. The Faculty of Technical Mathematics and Technical Physics at the Graz University of Technology supported my research indirectly by sponsoring the test drives and rental of optical speed sensors for one of my master students, Daniel Gander. His data are used in the experimental sections of this monograph, and he has also helped with additional experiments. The Institute of Automotive Engineering, in particular Klaus Prenninger and Erich Erhart helped with the optical speed sensors and provided a quick-start on vehicle dynamics. AVL Graz, in particular Urs Gerspach, is acknowledged for granting access to the fantastic AVL test track in Graz, where undisturbed test measurements could be carried out.

A calm home and a family which helps to see that life offers more than work, are the basis for success and contentment. I could not wish for a better family than the one I have. Resi, Valentin, and Valerie, I thank you for supporting me in any possible way, for helping me through intensive periods like the last few months, and for being there.

A.W.



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## Abbreviations and acronyms

C/N0	Carrier-to-noise-density ratio
CODE	Center for Orbit Determination in Europe
DD	Double-difference (observations)
DoD	US Department of Defense
DOP	Dilution of precision
ECEF	Earth-centered, earth-fixed
FLL	Frequency-lock-loop
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
IERS	International Earth Rotation and Reference System Service
IGMS	Engineering Geodesy and Measurement Systems (TUG)
IGS	International GNSS Service
IMU	Inertial measurement unit
KF	Kalman Filter
LOS	Line of sight
LOM	Local overall model test
MDB	Minimum detectable bias
PAYD	Pay-as-you-drive (insurance)
PLL	Phase-lock-loop
PV	Position/velocity
ppm	parts per million
PPS	Precise Positioning Service
RMS	Root mean square
RTK	Real-time kinematic (GPS)
SIS	Signal-in-space
STD	Standard deviation
SPS	Standard Positioning Service
TEC	Total electron content
TUG	Graz University of Technology
UD	Undifferenced (observations)
VTEC	Vertical TEC
WGS84	World Geodetic System 1984
ZPD	Zenith path delay

## Mathematical symbols

$a$	... scalar quantity
$\mathbf{a}$	... vector
$\mathbf{A}$	... matrix
$\mathbf{I}_n$	... $n \times n$ identity matrix
$\mathbf{0}$	... all-zero vector or all-zero matrix
$ a $	... absolute value of $a$
$\ \mathbf{a}\ $	... length of $\mathbf{a}$ (Euclidean norm)
$[a_{ij}]$	... the matrix with elements $a_{ij}$
$\mathbf{A}^T$	... transpose of $\mathbf{A}$
$\mathbf{x}_j^a$	... vector $\mathbf{x}_j$ expressed in the a-frame (coordinate frame "a")
$\mathbf{C}_a^b$	... a-frame to b-frame rotation matrix
$\dot{a}, \dot{\mathbf{a}}, \dot{\mathbf{A}}$	... first derivative with respect to time
$\ddot{a}, \ddot{\mathbf{a}}, \ddot{\mathbf{A}}$	... second derivative with respect to time
$a^{(n)}(t)$	... $n$ -th derivative of $a$ with respect to time
$\text{vec } \mathbf{A}$	... operator which stacks the columns of $\mathbf{A}$ beneath each other
$\Gamma(z)$	... Gamma function
$a \hat{=} b$	... $a$ corresponds to $b$ (e.g., equal except for units)
$a \lesssim b$	... $a$ is lower than or approximately equal to $b$
$\mathbf{a} \times \mathbf{b}$	... cross product of vectors $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{A} \otimes \mathbf{B}$	... Kronecker-Zehfuß product of $\mathbf{A}$ and $\mathbf{B}$
$E\{\dots\}$	... expected value
$D\{\dots\}$	... dispersion (variance-covariance matrix)
$\mathbf{x} \sim N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	... the elements of the $n \times 1$ vector $\mathbf{x}$ are normally distributed with expectation $\boldsymbol{\mu}$ and dispersion $\boldsymbol{\Sigma}$